I hated every minute of pain, but I said,
'Don't quit. Suffer now and live the rest of your life as a Champion'.

01. RELATION & FUNCTIONS

Q01. Relation R in the set A = {1, 2, 3} is as follows :
R = {(1,1), (2,2), (3,3), (1,3)}.
Write the ordered pairs to be added to R to make it the smallest equivalence relation.

Q02. Let A = {a, b, c} and the relation R be defined on A as follows :
R = {(a, a), (b, c), (a, b)}. Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Q03. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g (x) = $\alpha x + \beta$, then what value should be assigned to α and β?

Q04. Let $f : R \to R$ be the function defined by $f(x) = \dfrac{1}{2-\cos x}\ \forall x \in R$. Then find the range of f.

Q05. Let R denotes the equivalence relation in the set Z of integers given by R = {(a,b) : 2 divides a – b}. Write the equivalence class [0].

Q06. If A = {1, 2, 3} and f, g are relations corresponding to the subset of A × A indicated against them, which of f, g is a function? Why?
f = {(1, 3), (2, 3), (3, 2)}, g = {(1, 2), (1, 3), (3, 1)}.

Q07. If A = {a, b, c, d} and f = {a, b), (b, d), (c, a), (d, c)}, show that f is one-one from A onto A. Also find f^{-1}.

Q08. In the set of natural numbers N, define a relation R as follows: For all n, m ∈ N, nRm if on division by 5 each of the integers n and m leaves the remainder less than 5, i.e. one of the numbers 0, 1, 2, 3 and 4. Show that R is equivalence relation. Also, obtain the pair-wise disjoint subsets determined by R.

Q09. Show that the function $f : R \to R$ defined by $f(x) = \dfrac{x}{x^2+1}, \forall\ x \in R$, is neither one-one nor onto.

Q10. Let A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A × A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].

Q11. Let f, g : R → R be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x\ \forall\ x \in R$. Then, find fog and gof.

Q12. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is reflexive, transitive but not symmetric. True/False?

Q13. Set A has 3 elements and the set B has 4 elements. Then find the number of injective mappings that can be defined from A to B.

Q14. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Write the pre-images of 17 and – 3.

Q15. Real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is reflexive. True/False?

Q16. Consider the set A containing n elements. Find the total number of injective functions from A onto itself.

Q17. If the set A contains 5 elements and the set B contains 6 elements, then find the number of one-one and onto mappings from A to B.

Q18. Let A = {1, 2, 3, ..., n} and B = {a, b}. Write the number of surjections from A into B.

Q19. If $f(x) = 4 - (x-7)^3$, then find f^{-1}.

Q20. Let $f : R \to R$ be defined by $f(x) = \frac{x}{\sqrt{1+x^2}}$. Determine $(fofof)(x)$.

Q21. Determine the range of f if $f : [2, \infty) \to R$ be defined as $f(x) = x^2 - 4x + 5$.

02. INVERSE TRIGONOMETRIC FUNCTIONS

Q01. Find the principal value of $\cos^{-1}x$, for $x = \frac{\sqrt{3}}{2}$.

Q02. Evaluate: (a) $\tan^{-1}\sin\left(-\frac{\pi}{2}\right)$ (b) $\sin^{-1}\cos\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (c) $\sin 2\cot^{-1}(-5/12)$.

Q03. Find the value of: (a) $\tan^{-1}\tan\left(\frac{9\pi}{8}\right)$ (b) $\tan[\tan^{-1}(-4)]$ (c) $\sec\tan^{-1}\left(\frac{y}{2}\right)$.

Q04. Prove that $\tan(\cot^{-1}x) = \cot(\tan^{-1}x)$. State with reason, whether the equality is valid for all values of x?

Q05. Find the value of $\tan(\cos^{-1}x)$. Hence evaluate $\tan\cos^{-1}(8/17)$.

Q06. What is the value of $\cos[\sin^{-1}(1/4) + \sec^{-1}(4/3)]$?

Q07. Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

Q08. Find the value of $\sin[2\tan^{-1}(2/3)] + \cos(\tan^{-1}\sqrt{3})$.

Q09. Solve the equation: $\sin^{-1}6x + \sin^{-1}6\sqrt{3}x = -\frac{\pi}{2}$.

Q10. Evaluate: (a) $\sin^{-1}\cos\left(\frac{43\pi}{5}\right)$ (b) $\cos^{-1}[\cos(-680^\circ)]$.

Q11. If $\tan^{-1}x = \frac{\pi}{10}$ for some $x \in R$, then write the value of $\cot^{-1}x$.

Q12. Write the domain of: (a) $\sin^{-1}2x$ (b) $\sin^{-1}(-x^2)$ (c) $\cos^{-1}(x^2 - 4)$ (d) $\sin^{-1}x + \cos x$.

Q13. Determine the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.

Q14. Let $A = \sin^{-1}[\sin(-600^\circ)]$. Write the value of A.

Q15. Write the value of (a) $\sin[2\sin^{-1}(0.6)]$ (b) $\sin[2\tan^{-1}(.75)]$.

Q16. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\frac{1}{\sqrt{3}}$ has two solutions. True/ False?

Q17. If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$ then, write the values of α and β.

Q18. Evaluate $\tan^2\sec^{-1}2 + \cot^2\csc^{-1}3$.

Q19. Find the real solutions of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$.

Q20. If $2\tan^{-1}\cos A = \tan^{-1}(2\operatorname{cosec} A)$ then, show that $A = \frac{\pi}{4}$.

Q21. Show that $\tan\left[\frac{1}{2}\sin^{-1}\frac{3}{4}\right] = \frac{4-\sqrt{7}}{3}$ and, justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?

Q22. If $a_1, a_2, a_3, ..., a_n$ is an arithmetic progression with common difference d, then evaluate the following expression:

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+...+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right].$$

Q23. What is the domain of: (a) $\cos^{-1}(2x-1)$ (b) $\sin^{-1}\sqrt{x-1}$.

Q24. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then what is the value of $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$?

❖ State **True** or **False** for the statement in each of the Q25 to Q30:

Q25. All trigonometric functions have inverse over their respective domains.

Q26. The value of the expression $(\cos^{-1} x)^2$ is equal to $\sec^2 x$.

Q27. The domain of trigonometric functions can be restricted to any one of their branch (not necessarily principal value) in order to obtain their inverse functions.

Q28. The least numerical value, either positive or negative of angle θ is called principal value of the inverse trigonometric function.

Q29. The graph of inverse trigonometric function can be obtained from the graph of their corresponding trigonometric function by interchanging x and y axes.

Q30. The minimum value of n for which $\tan^{-1}(n/\pi) > \pi/4$, $n\ \pi \in N$, is valid is 5.

03. MATRICES & DETERMINANTS

Q01. Construct a matrix $A = [a_{ij}]_{2\times2}$ whose elements a_{ij} are given by $a_{ij} = e^{2ix}\sin jx$.

Q02. Show that a matrix which is both symmetric and skew symmetric is a zero matrix.

Q03. If A is 3 × 3 invertible matrix, then show that for any scalar k (non-zero), kA is invertible and $(kA)^{-1} = \frac{1}{k}A^{-1}$.

Q04. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A^2 - 4A + 7I = O$. Using this result, calculate A^5 also.

Q05. If A and B are square matrices of the same order, then what is the value of (A + B) (A – B)?

Q06. If A and B are symmetric matrices of the same order, then $(AB' - BA')$ is a skew symmetric matrix. True/ False?

Q07. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Q08. Prove by Mathematical Induction that $(A')^n = (A^n)'$, where $n \in N$ for any square matrix A.

Q09. If A is square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

Q10. If AB = BA for any two square matrices, prove by using principle of mathematical induction that, $(AB)^n = A^nB^n$.

Q11. Find x, y, z if $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfies $A' = A^{-1}$.

Q12. Find the total number of possible matrices of order 3 × 3 with each entry being 2 or 0.

Q13. If A and B are two matrices of the order 3 × m and 3 × n, respectively, and m = n, then what is the order of matrix (5A – 2B)?

Q14. If A is matrix of order m × n and B is a matrix such that AB′ and B′A are both defined, then what is the order of matrix B?

Q15. If A is a square matrix such that $A^2 = I$, then evaluate $(A - I)^3 + (A + I)^3 - 7A$.

Q16. If $\Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$, then prove that $\Delta_1 + \Delta_2 = 0$.

Q17. Prove that: (a) $\begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix} = 0$ (b) $\begin{vmatrix} 1 & 1 & 1 \\ {}^{n}C_1 & {}^{n+2}C_1 & {}^{n+4}C_1 \\ {}^{n}C_2 & {}^{n+2}C_2 & {}^{n+4}C_2 \end{vmatrix} = 8$.

Q18. Find the value of θ, satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$.

Q19. Let f (x) = $\begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$ then, evaluate: $\lim_{x \to 0} \frac{f(x)}{x^2}$.

Q20. If it is given that A and B are two skew-symmetric matrices of same order, then AB is symmetric matrix if _______.

Q21. The sum of the products of elements of any row with the co-factors of corresponding elements is equal to _________.

❖ State **True** or **False** for the statement in each of the Q22 to Q40:

Q22. A matrix denotes a number.

Q23. Matrices of any order can be added.

Q24. Two matrices are equal if they have same number of rows and same number of columns.

Q25. Matrices of different order can not be subtracted.

Q26. Matrix addition is associative as well as commutative.

Q27. Matrix multiplication is commutative.

Q28. A square matrix where every element is unity is called an identity matrix.

Q29. If A and B are two square matrices of the same order, then A + B = B + A.

Q30. If A and B are two matrices of the same order, then A – B = B – A.

Q31. If matrix AB = O, then A = O or B = O or both A and B are null matrices.

Q32. Transpose of a column matrix is a column matrix.

Q33. If A and B are two square matrices of the same order, then AB = BA.

Q34. If each of the three matrices of the same order are symmetric, then their sum is a symmetric matrix.

Q35. If A and B are any two matrices of the same order, then (AB)′ = A′B′.

Q36. If (AB)′ = B′A′, where A and B are not square matrices, then number of rows in A is equal to number of columns in B and number of columns in A is equal to number of rows in B.

Q37. If A, B and C are square matrices of same order, then AB = AC always implies that B = C.

Q38. AA′ is always a symmetric matrix for any matrix A.

Q39. If A is skew symmetric matrix, then A^2 is a symmetric matrix.

Q40. $(AB)^{-1} = A^{-1} B^{-1}$, where A and B are invertible matrices satisfying commutative property with respect to multiplication.

04. CONTINUITY & DIFFERENTIABILITY

Q01. If $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at x = 2, find the value of k.

Q02. Given f (x) = 1/ (x – 1). Find the points of discontinuity of composite function y = f [f (x)].

Q03. Let f (x) = x |x| , for all x ∈ R. Discuss the derivability of f (x) at x = 0.

Q04. If f (x) = |cos x|, find $f'\left(\dfrac{3\pi}{4}\right)$.

Q05. If f (x) = |cos x – sinx|, find $f'\left(\dfrac{\pi}{6}\right)$.

Q06. The function f (x) = 1/(x – [x]) is discontinuous for all x ∈ Z. True/ False?

Q07. Write the set of points where the function f given by f (x) = |x – 3|cos x is differentiable.

Q08. Write the number of points at which f (x) = 1/ log|x| is discontinuous.

Q09. Examine the continuity of the function $f(x) = \begin{cases} |x-a| \sin \dfrac{1}{x-a}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0.

Q10. Find all points of discontinuity of the function f (t) = 1/ (t^2 + t – 2), where t = 1/ (x–1).

Q11. Show that the function f (x) = |sin x + cos x| is continuous at x = π.

Q12. Differentiate log[log (log x^5)] with respect to x.

Q13. Find dy/dx, if $x = e^{\theta}(\theta + 1/\theta)$ and $y = e^{-\theta}(\theta - 1/\theta)$.

Q14. If x = a sin2t (1 + cos2t) and y = b cos2t (1 – cos2t), show that $\left(\dfrac{dy}{dx}\right)_{at\ t=\pi/4} = \dfrac{b}{a}$.

Q15. Find the values of p and q, so that $f(x) = \begin{cases} x^2 + 3x + p, & \text{if } x \leq 1 \\ qx + 2, & \text{if } x > 1 \end{cases}$ is differentiable at x = 1.

Q16. Write the set of points where the function f given by f (x) = |2x – 1|sin x is differentiable.

05. APPLICATIONS OF DERIVATIVES

Q01. For the curve y = 5x – $2x^3$, if x increases at the rate of 2units/sec, then how fast is the slope of curve changing when x = 3?

Q02. Water is dripping out from a conical funnel of semi-vertical angle π/4 at the uniform rate of 2cm^2/sec in the surface area, through a tiny hole at the vertex of the bottom. When the slant height of cone is 4cm, find the rate of decrease of the slant height of water.

Q03. Find the angle of intersection of the curves $y^2 = x$ and $x^2 = y$.

Q04. Prove that the function f (x) = tan x – 4x is strictly decreasing on (–π/3, π/3).

Q05. Show that the function f (x) = $4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

Q06. Using differentials, find the approximate value of $\sqrt{.082}$.

Q07. Find the condition for the curves $x^2/a^2 - y^2/b^2 = 1$; $xy = c^2$ to intersect orthogonally.

Q08. Show that the local maximum value of x + 1/x is less than local minimum value.

Q09. Water is dripping out at a steady rate of 1 cu. cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\pi/6$.

Q10. Find the angle of intersection of the curves $y^2 = 4ax$ and $x^2 = 4by$.

Q11. Find the maximum and minimum values of $f(x) = \sec x + \log \cos^2 x$, $0 < x < 2\pi$.

Q12. Find the area of greatest rectangle that can be inscribed in an ellipse with major axis along x-axis and centre at the origin.

Q13. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on the interval $[-\pi/2, \pi/2]$.

Q14. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \pi/6$.

Q15. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

Q16. A kite is moving horizontally at a height of 151.5meters. If the speed of kite is 10m/s, how fast is the string being let out; when the kite is 250m away from the boy who is flying the kite? The height of boy is 1.5m.

Q17. Two men A and B start with velocities v at the same time from the junction of two roads inclined at 45° to each other. If they travel by different roads, find the rate at which they are being separated.

Q18. Find the approximate volume of metal in a hollow spherical shell whose internal and external radii are 3cm and 3.0005cm, respectively.

Q19. A man, 2m tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{3}$ m from the base of the light?

Q20. A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and $L = 200(10 - t)^2$. How fast is the water running out at the end of 5seconds? What is the average rate at which the water flows out during the first 5 seconds?

Q21. Let x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

Q22. Show that $f(x) = 2x + \cot^{-1}x + \log(\sqrt{1+x^2} - x)$ is increasing in R.

Q23. A telephone company in a town has 500 subscribers on its list and collects fixed charges of ₹300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of ₹1/- one subscriber will discontinue the service. Find what increase will bring maximum profit?

Q24. If the straight line $x \cos\alpha + y \sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

Q25. If the sum of the surface areas of cube and a sphere is constant, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum?

Q26. AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles.

Q27. A metal box with a square base and vertical sides is to contain $1024cm^3$. The material for the top and bottom costs ₹5/cm^2 and the material for the sides costs ₹2.50/cm^2. Find the least cost of the box.

Q28. The sum of the surface areas of a rectangular parallelopiped with sides x, 2x and x/3 and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.

06. INTEGRAL CALCULUS [INDEFINITE & DEFINITE INTEGRALS]

Q01. Integrate $\frac{2a}{\sqrt{x}}-\frac{b}{x^2}+3c\sqrt[3]{x^2}$ w.r.t. x.

Q02. Evaluate: $\int \tan^8 x\sec^4 x\,dx$.

Q03. Write the value of: (a) $\int \frac{x^3}{x^4+3x^2+2}dx$ (b) $\int \frac{dx}{2\sin^2 x+5\cos^2 x}$.

Q04. Find the value of: $\int x^2\tan^{-1}x\,dx$.

Q05. Evaluate: $\int \frac{x^2}{x^4+x^2-2}dx$.

Q06. What is the integral value of $\sin^2 x/(\sin x+\cos x)$ within the limits $x=0$ to $x=\pi/2$?

Q07. Find the value of the integral: $\int_0^1 x\,(\tan^{-1}x)^2\,dx$.

Q08. Evaluate $\int_{-1}^{2} f(x)dx$, where $f(x) = |x+1| + |x| + |x-1|$.

Q09. Is $\int_{a+c}^{b+c} f(x)dx$ equal to $\int_a^b f(x+c)\,dx$? Yes/No?

Q10. If f and g are continuous functions in [0, 1] satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = a$, then evaluate: $\int_0^a f(x).g(x)dx$.

Q11. If $\int_0^y \frac{dx}{\sqrt{1+9t^2}}$ and $d^2y/dx^2 = ay$ then, determine the value of a.

Q12. The value of the definite integral $\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx$ is 2 log(2). True/ False?

Q13. If $\int_0^1 \frac{e^t}{1+t}dt=a$, then write the value of $\int_0^1 \frac{e^t}{(1+t)^2}dt$.

Q14. Evaluate: a) $\int_{-2}^{2}|x\cos\pi x|dx$ (b) $\int_0^7 \frac{\sin^6 x}{\cos^8 x}dx$.

Q15. Verify the following integrals:

(a) $\int \frac{2x-1}{2x+3}dx = x-\log\left|(2x+3)^2\right|+C$ (b) $\int \frac{2x+3}{x^2+3x}dx=\log|x^2+3x|+C$.

Q16. Evaluate the following indefinite integrals:

(a) $\int \tan^2 x\sec^4 x\,dx$ (b) $\int \frac{x\,dx}{\sqrt{x}+1}$ (c) $\int \frac{x^{1/2}dx}{1+x^{3/4}}$

(d) $\int \frac{\sqrt{1+x^2}}{x^4}dx$ (e) $\int \frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$ (f) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$

(g) $\int \frac{\cos x-\cos 2x}{1-\cos x}dx$ (h) $\int \frac{dx}{x\sqrt{x^4-1}}$ (i) $\int \frac{x+\sin x}{1+\cos x}dx$.

Q17. Evaluate the following definite integrals:

(a) $\int_0^{\pi/2} \frac{\tan x\,dx}{1+m^2\tan^2 x}$ (b) $\int_0^{1/2} \frac{dx}{(1+x^2)\sqrt{1-x^2}}$ [Hint: Put $x=\sin\theta$]

(c) $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}\,dx}{(1-\cos x)^{5/2}}$ (d) $\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x+b^2\sin^2 x)^2}$ [Hint: Divide Nr&Dr by $\cos^4 x$]

(e) $\int_0^{\pi} x\log\sin x\,dx$ (f) $\int_{-\pi/4}^{\pi/4} \log(\sin x+\cos x)dx$.

Q18. Evaluate: $\int \frac{x^2dx}{x^4-x^2-12}$.

07. APPLICATION OF INTEGRALS

Q01. Find the area of the region bounded by the curve $ay^2 = x^3$, the y-axis, y = a and y = 2a.
Q02. Find the area enclosed by the curve x = 3 cos t and y = 2 sin t.
Q03. Find the area of the region bounded by the curves $x = at^2$ and y = 2at between the ordinate corresponding to t = 1 and t = 2.
Q04. The area of the region bounded by the curve $y = x^2 + x$, x-axis and the line x = 2 and x = 5 is equal to 297/6 sq.units. True/False?
Q05. Find the area of the region bounded by the curve $y = x^3$, y = x + 6 and x = 0.
Q06. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines x = 0 and x = 1.

08. DIFFERENTIAL EQUATIONS

Q01. Find the differential equation of all non-horizontal lines lying in a plane.
Q02. Write the degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$.
Q03. The solution of differential equation $2x\,y_1 - y = 3$ represents a family of parabolas. Yes/No?
Q04. Write the integrating factor of (dy/dx) (x log x) + y = 2 log x.
Q05. Show that $F(x, y) = \frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ a homogeneous function. What is its degree?
Q06. Solve: (x + y) (dx – dy) = dx + dy.
Q07. Write the integrating factor of the difeerential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$.

09. VECTOR ALGEBRA

Q01. If the points (–1, –1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of m.

Q02. If $\vec{a}$ and $\vec{b}$ are unit vectors then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a}-\vec{b}$ to be a unit vector?

Q03. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B respectively then, find the position vector of a point C in BA produced such that BC = 1.5 BA.

Q04. The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ is two. True/False?

Q05. When will the vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$?

10. THREE DIMENSIONAL GEOMETRY

Q01. The x-coordinate of a point on the line joining the points Q(2, 2, 1) and R(5, 1, –2) is 4. Find its z-coordinate.

Q02. Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

Q03. Find the coordinates of the point where the line through (3, – 4, – 5) and (2, –3, 1) crosses the plane passing through three points (2, 2, 1), (3, 0, 1) and (4, –1, 0).

Q04. Find the distance of the point (–1, –5, – 10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+p(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$.

Q05. Find the angle between the lines whose direction cosines are given by following equations:

$$3l + m + 5n = 0 \text{ and } 6mn - 2nl + 5lm = 0.$$

Q06. The coordinates of the foot of perpendicular drawn from the point (2, 5, 7) on the x-axis are given by (2, 0, 0). True/False?

Q07. Write the equation of x-axis in space.

Q08. Show that the points A(1, 2, 3), B(–2, 3, 4) and C(7, 0, 1) are collinear.

Q09. Check if the lines $\vec{r}=3\hat{i}-2\hat{j}+6\hat{k}+\lambda(2\hat{i}+\hat{j}+2\hat{k})$ and $\vec{r}=2\hat{j}-5\hat{k}+\mu(6\hat{i}+3\hat{j}+2\hat{k})$ intersect each other? If they intersect then, find the angle at which they cut each other.

Q10. Find the equation of a plane which bisects perpendicularly the line joining the points given as A(2, 3, 4) and B(4, 5, 8) at right angles.

Q11. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axis.

Q12. If the line drawn from the point *say* P(–2, – 1, – 3) meets a plane at right angle at the point Q(1, – 3, 3) then, find the equation of the plane.

Q13. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle of 60^{o} each.

Q14. Find the angle between the lines whose direction cosines are given by the equations:

$$l + m + n = 0,\ l^2 + m^2 - n^2 = 0.$$

Q15. If a variable line in two adjacent positions has direction cosines l, m, n and $l+\delta l$, $m+\delta m$, $n+\delta n$, show that the small angle $\delta\theta$ between the two positions is given by

$$\delta\theta^2 = \delta l^2 + \delta m^2 + \delta n^2.$$

Q16. O is the origin and A is (a, b, c). Find the direction cosines of the line OA and the equation of plane through A at right angle to OA.

Q17. Two systems of rectangular axis have the same origin. If a plane cuts them at distances a, b, c and a′, b′, c′, respectively, from the origin, prove that:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}.$$

Q18. Find the equations of line passing through the point (3, 0, 1) and parallel to the planes $x+2y=0$ and $3y-z=0$.

Q19. The plane $x - y - z = 4$ is rotated through an angle of 90^o about its line of intersection with the plane $x + y + 2z = 4$. Find its equation in the new position.

Q20. The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is given by $ax+by\pm\left(\sqrt{a^2+b^2}\tan\alpha\right)z=0$.

Q21. A mirror and a source of light are situated at the origin O and at a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the direction ratios of the normal plane are 1, –1, 1 then, find the direction cosines of the reflected ray.

Q22. Show that the points (1,–1, 3) and (3, 3, 3) are equidistant from the plane $5x + 2y - 7z + 9 = 0$ and lies on the opposite side of it.

Q23. Show that the straight lines whose direction cosines are given by $2l + 2m - n = 0$ and $mn+nl+lm=0$ are at right angles.

Q24. If l_1, m_1, n_1; l_2, m_2, n_2; l_3, m_3, n_3 are the direction cosines of three mutually perpendicular lines, then prove that the line whose direction cosines are proportional to $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3$, $n_1 + n_2 + n_3$ makes equal angles with them.

11. LINEAR PROGRAMMING PROBLEMS

Q01. Corner points of the feasible region determined by a system of linear constraints are found to be (0, 10), (5, 5), (15, 15), (0, 20). Let $Z = p x + q y$, where $p, q > 0$. Write the condition(s) on p and q so that the maximum of Z occurs at both the points (15, 15) and (0, 20).

Q02. An oil company requires 13000, 20000, and 15000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300, and 200 barrels per day of high, medium and low grade oil respectively whereas Refinery B produces 200, 400, and 100 barrels per day respectively. If A costs Rs.400 per day and B costs Rs.300per day to operate, how many days should each refinery be run to minimize the cost of meeting-requirements?

Q03. In order to supplement daily diet, a person wishes to take some X and some Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligrams per tablet) are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs at least 18mg of iron, 21mg of calcium and 16mg of vitamins. The price of each tablet of X and Y is Rs.2 and Re.1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

Q04. Vishu has two courses to prepare for final examination. Each hour of study, he devotes to course A is expected to return Rs.600 in terms of long range job benefits. Each hour devoted to course B is expected to return Rs.300 in terms of long range job benefits. The stores are closed and Vishu has only 15 chewing gums. He finds that he consumes one chewing gums every 20 minutes while studying course B and every 12 minutes while studying course A. Time is running short only four hours are left to prepare. Vishu feels that he must devote at least 2 hours to study. Using linear programming determine an optimal policy for Vishu that would maximize his return in terms of long range job benefits.

Q05. A catering agency has two kitchens to prepare food at two places A and B. From these places **Mid Day Meal** is to be supplied to three different schools situated at P, Q and R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacities of kitchens A and B are 60 and 70 packets respectively. The transportation cost per packet from the kitchens to schools is given below:

Transportation cost per packet (in Rs.)

To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to the schools so that the cost of transportation is minimum? Also find the minimum cost. **While transportation of food packets from the kitchens, 10 packets got stolen. You know the thief, who is a poor friend of yours. How would you inform the kitchen owner of the theft and, at the same time saving your friend?**

Q06. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs.50 and that of type B circuit is Rs.60, formulate this as a linear programming problem so that the manufacturer can maximize his profit. Also solve the L.P.P. graphically to find the maximum profit. **The owner of this manufacturing unit is knowingly producing defective circuitries with an aim of earning more money. How would you stop him doing that by making him conscious of his wrong act?**

Q07. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs.2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs.3 per km. He has at most Rs.120 to spend on petrol and one hour's time. Find the maximum distance that he can travel using LPP.

10. PROBABILITY – THE THEORY OF CHANCES

Q01. In a binomial distribution, the sum of mean and variance of 5 trials is known to be 3.75. Find the distribution.

Q02. The probability of simultaneous occurrence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q, prove that P (A′) + P (B′) = 2 – 2p + q.

Q03. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red in colour.

Q04. Two dice are thrown together. Let A be the event 'getting 6 on the first die' and B be the event 'getting 2 on the second die'. Are the events A and B independent?

Q05. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

Q06. Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws.

Q07. In a dice game, a player pays a stake of Re.1 for each throw of a die. She receives Rs.5 if the die shows a 3, Rs.2 if the die shows a 1 or 6, and nothing otherwise. What is the player's expected profit per throw over a long series of throws?

Q08. Suppose 10,000 tickets are sold in a lottery each for Re.1. First prize is of Rs.3000 and the second prize is of Rs.2000. There are three third prizes of Rs.500 each. If you buy one ticket, what is your expectation?

Q09. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Q10. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then another ball is drawn at random. What is the probability of second ball being blue?

Q11. Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

Q12. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Q13. The random variable X can take only the values 0, 1, 2. Given that P(X = 0) = P (X = 1) = p and that $E(X^2) = E(X)$, find the value of p.

Q14. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. It A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

Q15. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k.

Q16. Three bags contain a number of red and white balls as follows:
Bag I : 3 red balls, Bag II : 2 red balls and 1 white ball, Bag III : 3 white balls.
The probability that bag i will be chosen and a ball is selected from it is i/6, where i = 1, 2, 3. What is the probability that (i) a red ball will be selected? (ii) a white ball is selected?

Q17. Refer to Previous Question. If a white ball is selected, what is the probability that it came from (i) Bag II (ii) Bag III.

Q18. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability
(i) of a randomly chosen seed to germinate,
(ii) that it will not germinate given that the seed is of type A_3,
(iii) that it is of the type A_2 given that a randomly chosen seed does not germinate.

Q19. A letter is known to have come either from TATA NAGAR or from CALCUTTA. On the envelope, just two consecutive letter TA are visible. What is the probability that the letter came from TATA NAGAR?

Q20. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the 1st bag; but it shows up any other number, a ball is chosen from the second bag. Find the probability of choosing a black ball.

Q21. Let X be a discrete random variable whose probability distribution is defined as follows:

$$P(X=x)=\begin{cases} k(x+1), & \text{for } x=1, 2, 3, 4 \\ 2kx, & \text{for } x=5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Calculate
(i) the value of k (ii) E (X) (iii) Standard deviation of X.

Q22. The probability distribution of a random variable x is given as under:

$$P(X=x)=\begin{cases} kx^2, & \text{for } x=1, 2, 3 \\ 2kx, & \text{for } x=4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Calculate
(i) E(X) (ii) $E(3X^2)$ (iii) $P(X \geq 4)$.

Q23. A bag contains (2n + 1) coins. It is known that n of these coins have a head on both sides where as the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is 31/42, determine the value of n.

HINTS & ANSWERS
(ADVANCED LEVEL QUESTIONS)

01. RELATION & FUNCTIONS

Q01. (3,1) Q02. (b, b), (c, c), (a, c) Q03. 2, –1 Q04. $\left[\frac{1}{3},1\right]$

Q05. $[0]=\{0,\pm 2,\pm 4,\pm 6,\ldots\}$ Q06. f is a function since each element of A in the first place in the ordered pairs is related to only one element of A in the second place while g is not a function because 1 is related to more than one element of A, namely, 2 and 3.

Q07. f is one-one since each element of A is assigned to distinct element of the set A. Also, f is onto since f (A) = A. Moreover, f^{-1} = {(b, a), (d, b), (a, c), (c, d)}.

Q08. R is reflexive since for each a ∈ N, aRa. R is symmetric since if aRb, then bRa for a, b ∈ N. Also, R is transitive since for a, b, c ∈ N, if aRb and bRc, then aRc. Hence R is an equivalence relation in N which will partition the set N into the pair-wise disjoint subsets. The equivalent classes are given as : A_0 = {5, 10, 15, 20 ...}, A_1 = {1, 6, 11, 16, 21 ...}, A_2 = {2, 7, 12, 17, 22, ...}, A_3 = {3, 8, 13, 18, 23, ...}, A_4 = {4, 9, 14, 19, 24, ...}. It is evident that the above five sets are pair-wise disjoint and $A_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 = \bigcup_{i=0}^{4} A_i = N$.

Q09. For $x_1, x_2 \in R$, consider $f(x_1) = f(x_2)$ and obtain $x_1 = x_2$ or $x_1x_2 = 1$.
We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, for instance, if we take $x_1 = 2$ and $x_2 = 1/2$, then we have $f(x_1) = 2/5$ and $f(x_2) = 2/5$ but $2 \neq 1/2$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R\ \exists\ x \in R$ such that $f(x) = 1$ which gives $\frac{x}{x^2+1}=1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

Q10. [(2, 5)] = {(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)}

Q11. $(fog)(x)=\begin{cases} 0, x \geq 0 \\ -4x, x<0 \end{cases}$, $(gof)(x)=0\ \forall x \in R$ Q12. True

Q13. The total number of injective mappings from the set containing 3 elements into the set containing 4 elements is $^4P_3 = 4! = 24$. Q14. {4, –4}, ϕ

Q15. True Q16. n! Q17. 0 Q18. $2^n - 2$

Q19. $7 + (4 - x)^{1/3}$ Q20. $\frac{x}{\sqrt{1+3x^2}}$ Q21. $[1,\infty)$

02. INVERSE TRIGONOMETRIC FUNCTIONS

Q01. $\frac{\pi}{6}$ Q02. a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) –120/169

Q03. a) $\frac{\pi}{8}$ (b) –4 (c) $\frac{\sqrt{4+y^2}}{2}$

Q04. Let $\cot^{-1}x = \theta$. Then $\cot\theta = x$ or, $\tan(\pi/2 - \theta) = x$ ⇒ $\tan^{-1}x = \pi/2 - \theta$.
So $\tan(\cot^{-1}x) = \tan\theta = \cot(\pi/2 - \theta) = \cot(\pi/2 - \cot^{-1}x) = \cot(\tan^{-1}x)$.
The equality is valid for all values of x since $\tan^{-1}x$ and $\cot^{-1}x$ are true for all x ∈ R.

Q05. $\frac{\sqrt{1-x^2}}{x}, \frac{15}{8}$ Q06. $\frac{3\sqrt{15}-\sqrt{7}}{16}$ Q08. 37/26

Q09. –1/12 Q10. a) $-\frac{\pi}{10}$ (b) 40º Q11. $\frac{2\pi}{5}$ Q12. a) [–1/2, 1/2]

(b) $|x| \le 1$ i.e., $x \in [-1, 1]$ (c) $x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ (d) $x \in R \cap [-1,1]$ i.e., [–1, 1]

Q13. $5\pi^2/4$, $\pi^2/8$ Q14. $\pi/3$ Q15. a) 0.96 (b) 0.96

Q16. It has unique solution, which is $x = \sqrt{3}$. Q17. $\alpha = 0, \beta = \pi$

Q18. 11 Q19. 0, –1 Q22. $(a_n - a_1)/(1 + a_1 a_n)$

Q23. a) [0, 1] (b) [1, 2] Q24. 6 Q25. False Q26. False

Q27. True Q28. True Q29. True Q30. False.

03. MATRICES & DETERMINANTS

Q01. $\begin{bmatrix} e^{2x}\sin x & e^{2x}\sin 2x \\ e^{4x}\sin x & e^{4x}\sin 2x \end{bmatrix}$ Q04. $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$ Q05. $A^2 - AB + BA - B^2$ Q06. True

Q07. 28 × 1, 1 × 28, 4 × 7, 7 × 4, 14 × 2, 2 × 14. If matrix has 13 elements then its order will be either 13 × 1 or 1 × 13 Q11. $\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{6}}, \pm\frac{1}{\sqrt{3}}$ Q12. 512 Q13. 3 × n Q14. m × n

Q18. $\theta = n\pi$ or $n\pi + (-1)^n\left(\frac{\pi}{6}\right)$ Q19. 0 Q20. Self Q21. Self Q22. False

Q23. False Q24. False Q25. True Q26. True Q27. False Q28. False Q29. True

Q30. False Q31. False Q32. False Q33. False Q34. True Q35. False Q36. True

Q37. False Q38. True Q39. True Q40. True.

04. CONTINUITY & DIFFERENTIABILITY

Q01. 7 Q02. Points of discontinuity: x = 1, 2 Q03. Since the left hand derivative and right hand derivative both are equal, hence f is differentiable at x = 0.

Q04. When $\frac{\pi}{2} < x < \pi$, $\cos x < 0$ so that $|\cos x| = -\cos x$ i.e., $f(x) = -\cos x \Rightarrow f'\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Q05. $f'\left(\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} - \cos\frac{\pi}{6} = -\frac{1}{2}(1+\sqrt{3})$. Q06. True Q07. R – {3}

Q08. Discontinuity at X = –1, 0, 1. Hence total number of points of discontinuity = 3.

Q09. Continuous Q10. 1, 2, 1/2 Q12. $\frac{5}{x\log x^5 \log\ \log x^5}$

Q13. $e^{-2\theta}\left(\frac{1+\theta+\theta^2-\theta^3}{\theta^3+\theta^2+\theta-1}\right)$ Q15. p = 3, q = 5 Q16. R – {1/2}.

05. APPLICATIONS OF DERIVATIVES

Q01. Decreasing at the rate of 72units/sec Q02. $\frac{\sqrt{2}}{4\pi}$ cm/sec Q03. $\pi/2$, $\tan^{-1}(3/4)$

Q05. Show that the critical point (which is x = 3/2) is the point of inflection. Q06. 0.2867

Q07. $a^2 - b^2 = 0$ Q09. $\frac{1}{2\sqrt{3}\pi}$ cm/sec Q10. $\tan^{-1}\left(\frac{3a^{1/3}b^{1/3}}{2(a^{2/3}+b^{2/3})}\right)$

Q11. Max. Value: –1, 1 at $x = \pi$ and 0 respectively; Min. Value: 2(1 – log2), 2(1 – log2) at $x = \pi/3$ and $5\pi/3$ respectively. Q12. 2ab sq.units Q13. Difference: $\pi/2 - (-\pi/2) = \pi$.

Q16. 8m/sec Q17. $[\sqrt{2-\sqrt{2}}]v$ units/sec Q18. $0.018\pi\ cm^3$

Q19. $2\frac{2}{3}$ m/sec towards light, –1m/sec Q20. 2000 litres/sec, 3000 litres/sec

Q21. $2x^3 - 3x + 1$ Q23. `100 Q24. Let $P(x_1, y_1)$ be the point of contact. So it would satisfy the curve i.e., $b^2x_1^2 + a^2y_1^2 = a^2b^2$...(i). Slope of tangent at P : $-\frac{b^2x_1}{a^2y_1}$. Then, the equation of tangent at P is : $b^2x_1x + a^2y_1y = b^2x_1^2 + a^2y_1^2$ i.e., $b^2x_1x + a^2y_1y = a^2b^2$...(ii) [By using (i) But the given equation of tangent is $x\cos\alpha + y\sin\alpha = p$ …(iii). By (ii) and (iii), $\frac{b^2x_1}{\cos\alpha} = \frac{a^2y_1}{\sin\alpha} = \frac{a^2b^2}{p}$. That gives, $x_1 = \frac{a^2\cos\alpha}{p}$ and $y_1 = \frac{b^2\sin\alpha}{p}$. Substitute these values in (i) to get the desired result. Q25. 1:1 Q27. ₹1920 Q28. $\frac{2}{3}x^3\left(1+\frac{2\pi}{27}\right)$.

06. INTEGRAL CALCULUS – Indefinite & Definite Integrals

Q01. $4a\sqrt{x} + \frac{b}{x} + \frac{9c}{5}\sqrt[3]{x^5} + k$ Q02. $\frac{\tan^{11}x}{11} + \frac{\tan^9 x}{9} + k$ Q03.a) $\log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + k$

(b) $\frac{1}{\sqrt{10}}\tan^{-1}\left(\sqrt{\frac{2}{5}}\tan x\right) + k$ Q04. $\frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\log|1+x^2| + k$

Q05. Put $x^2 = y$ for the partial fraction $\Rightarrow \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + k$

Q06. $(1/\sqrt{2})\log(\sqrt{2}+1)$ Q07. $\frac{\pi^2-4\pi}{16} + \frac{1}{2}\log 2$ Q08. 19/2 Q09. Yes

Q10. $\frac{a}{2}\int_0^a f(x)dx$ Q11. a = 9 Q12. True Q13. a + 1 – (e/2)

Q14.a) $\frac{8}{\pi}$ (b) 1/7 Q15.a) Self (b) Self

Q16.a) $(\tan^5 x)/5 + (\tan^3 x)/3 + C$ (b) $2\left[\frac{x\sqrt{x}}{3} - \frac{x}{2} + \sqrt{x} - \log|\sqrt{x}+1|\right] + C$

(c) $\frac{4}{3}\left[x^{3/4} - \log|1+x^{3/4}|\right] + C$ (d) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2} + C$ (e) $-\frac{1}{2}\sin 2x \sin x + C$

(f) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C$ (g) 2 sin x + x + C (h) $(1/2)\sec^{-1}(x^2) + C$ (i) $x\tan\frac{x}{2} + C$

Q17.a) $\frac{\log m}{m^2-1}$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{\sqrt{2}}{3}$ (c) 3/2 (d) $\frac{\pi}{4}\left(\frac{a^2+b^2}{a^3b^3}\right)$

(e) $\frac{\pi^2}{2}\log\left(\frac{1}{2}\right)$ (f) $\frac{\pi}{4}\log\left(\frac{1}{2}\right)$ Q18. $\frac{1}{7}\log\left|\frac{x-2}{x+2}\right|+\frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}}+C$.

07. APPLICATION OF INTEGRALS

Q01. $\frac{3}{5}a^2\left|2^{5/3}-1\right|$ sq.units Q02. Note that it is the equation of ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, in parametric form. Req. Area = 6π sq.units Q03. Note that it is the equation of parabola $y^2=4ax$, in parametric form. Req. Area = $\frac{56}{3}a^2$ sq.units Q04. True

Q05. 10 sq.units Q06. 4/3 sq.units

08. DIFFERENTIAL EQUATIONS

Q01. The general equation of all non-horizontal lines in a plane is $ax + by = c \Rightarrow \frac{d^2x}{dy^2}=0$.

Q02. Not defined Q03. Yes Q04. log x Q05. Degree = 0

Q06. Substitute x + y = z after seperating dx and dy. Q07. e^x/x

09. VECTOR ALGEBRA

Q01. m = 8 Q02. 30^o Q03. $\frac{3\vec{b}-\vec{a}}{2}$

Q04. True Q05. When the vectors $\vec{a}$ and $\vec{b}$ are equal vectors.

10. THREE DIMENSIONAL GEOMETRY

Q01. Use section formula $\Rightarrow$ z coordinate : –1 Q02. Write the given point on the line *say* P(–3, 4, –8). Then determine the coordinate of any random point on the given line *say* Q(3m – 3, 5m + 4, 6m – 8). Write the d.r.'s of line PQ. Use the fact that PQ is $\perp^{er}$ to the given line, to find the value of m = 3/10. [In short, find the **foot of perpendicular**.] Hence, Required distance, PQ = $\sqrt{\frac{37}{10}}$units . Q03. (1, –2, 7) Q04. 13units Q05. $\cos^{-1}(-1/6)$ Q06. True

Q07. y = 0, z = 0 Q08. Show that the d.r.'s of lines AB and BC are proportional. By using the fact that only parallel lines have proportionate d.r.'s, the lines AB and BC are parallel. But since B is a common point, so the points A, B C must be collinear.

Q09. Yes. Angle = $\cos^{-1}(19/21)$ Q10. x + y + 2z = 19 Q11. x + y + z = 9

Q12. 3x – 2 y + 6z = 27 Q13. $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ and $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ Q14. 60^o

Q16. $ax+by+cz=a^2+b^2+c^2$ Q18. $(x-3)\hat{i}+y\,\hat{j}+(z-1)\hat{k}=\lambda(-2\hat{i}+\hat{j}+3\hat{k})$

Q19. Given planes are π_1 and π_2 *say*. Required plane is π_3: x + y + 2z – 4 + k (x – y – z – 4) = 0. Since the planes π_1 and π_3 are at right angles to each other so, $(1+k)-(1-k)-(2-k)=0$ $\Rightarrow k=\frac{2}{3}$. Hence, the required plane is: $5x+y+4z=20$. Q21. –1/3, –2/3, 2/3.

11. LINEAR PROGRAMMING PROBLEMS

Q01. Self Q02. Self Q03. Tablet X : 1, Tablet Y : 6

Q04. Self Q05. Self Q06. Self

Q07. To Maximize: $Z = x + y$. Subject to constraints: $2x + 3y \le 120$, $8x + 5y \le 400$, $x \ge 0$, $y \ge 0$.

12. PROBABILITY – THE THEORY OF CHANCES

Q01. Find $p = q = 1/2$. Then use $P(X = r) = C(n, r)\, p^r q^{n-r}$ where $n = 5$, $r = 0, 1, 2, 3, 4, 5$.

Q02. Since P(exactly one of A and B occurs) = q (given), we get

$P(A\cup B) - P(A\cap B) = q \Rightarrow p - P(A\cap B) = q \Rightarrow P(A\cap B) = p - q$

$\Rightarrow 1 - P(A'\cup B') = p - q \Rightarrow P(A'\cup B') = 1 - p + q \Rightarrow P(A') + P(B') - P(A'\cap B') = 1 - p + q$

$\Rightarrow P(A') + P(B') = (1 - p + q) + P(A'\cap B') = (1 - p + q) + [1 - P(A\cup B)] = (1 - p + q) + (1 - p)$

$\therefore \quad = 2 - 2p + q.$

Q03. 1/5 Q04. Events A and B are independent since $P(A \cap B) = P(A)\,P(B)$

Q05. 168/225 Q06. Find P(at least 8 successes) = P(8) + P(9) + P(10) = $201/3^{10}$

Q07. Rs.0.50 Q08. Rs.0.65 Q09. 85/153 Q10. 5/9

Q11. 1/3 Q12. 9/44 Q13. 1/2 Q14. 775/7776

Q16. 7/18, 11/18 Q17. 2/11, 9/11 Q18. 0.49, 0.65, 0.314

Q19. Let E_1, E_2 and E be the events defined as 'letter has come from CALCUTTA', 'letter has come from TATANAGAR' and 'two consecutive letters TA are visible on the envelope' respectively.

Now $P(E_1) = 1/2$, $P(E_2) = 1/2$, $P(E|E_1) = \dfrac{n(E \cap E_1)}{n(E_1)} = 1/7$

[As seven pairs of consecutive letters are CA, AL, LC, CU, UT, TT, TA],

and, $P(E|E_2) = \dfrac{n(E \cap E_2)}{n(E_2)} = 2/8$

[As eight pairs of consecutive letters are TA, AT, TA, AN, NA, AG, GA, AR].

Hence required probability, $P(E_2|E) = 7/11$.

Q20. 11/21 Q21. 1/50, 5.2, 1.7 (Approx.)

Q22. 4.32, 61.9, 15/22 Q23. 10.