Applications Of Derivatives

TEST – 01

Q01. Show that the normal at any point θ to the curve $x = k(\cos\theta + \theta\sin\theta)$, $y = k(\sin\theta - \theta\cos\theta)$ is at a constant distance from the origin.

Q02. What is the condition that the curves $ax^2 + by^2 = 1$ and $\alpha x^2 + \beta y^2 = 1$ may cut each other orthogonally?

OR Check if the curves $xy = 25$ and $x^2 + y^2 = 50$ touch each other or, not?

Q03. Show that the semi-vertical angle of a conical milk vessel of greatest capacity and of given slant height is $\tan^{-1}\sqrt{2}$. What is the importance of milk for growing children?

Q04. If the slope of curve $y = \dfrac{ax}{b-x}$ at $(1,1)$ is 2, find the values of a and b.

Q05. Prove that the line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ touches the curve $\dfrac{x}{a} + \log\left(\dfrac{y}{b}\right) = 0$ at the point where it crosses y-axis.

Q06. Find the intervals of increasing or decreasing for $f(x) = 2\log(x-2) - x^2 + 4x + 1$.

OR Determine the interval(s) in which x^x is increasing or decreasing.

Q07. The combined resistance R of two resistors is given by $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, where R_1 and R_2 are the respective resistances of two resistors with the condition $R_1 + R_2 = k$, $(R_1, R_2 > 0)$, where k is a constant. Show that the maximum resistance R is obtained by choosing resistor for which $R_1 = R_2$.

Q08. Show that the function given as $f(x) = \tan^{-1}(\sin x + \cos x)$, $x > 0$ is always a strictly increasing function in the open interval $\left(0, \dfrac{\pi}{4}\right)$.

Q09. Prove that the area of a right angled triangle of given hypotenuse is maximum when the triangle is isosceles. Also find the area of triangle.

Q10. A wire of length $20m$ is to be cut into two pieces. One of the pieces is bent to form a square and the other into an equilateral triangle. What should be the length of two pieces so that their combined area is as minimum as possible?

TEST – 02

Q01. At what point(s) is the line $x-y+1=0$ tangent to the curve $y=2x^2+3$?

Q02. Find the equation of normal to the curve $x^2=4y$ which passes through the point $(1,2)$.

Q03. Prove that the curves $x=y^2$ and $xy=m$ cut each other orthogonally if $8m^2=1$.

Q04. Find the intervals in which followings are increasing or decreasing:

(a) $f(x)=\sin x+\cos x,\ 0<x<2\pi$ (b) $f(x)=2x^3-15x^2+36x+1$.

Q05. Find approximate value of $\sqrt{.037}$ using derivatives.

Q06. Find the volume of largest cylinder that can be inscribed in a sphere of radius R.

Q07. A ladder $20m$ long has one end on the ground and the other end in contact with a vertical wall. The lower end slips along the ground. Show that when the lower end of the ladder is $16m$ away from the wall, the upper end is moving $\frac{4}{3}$ times as faster as the lower end.

OR Sand is pouring from a pipe at the rate of $12cm^3/sec$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of its base. How fast is the height of sand-cone increasing when the height is $4cm$?

Q08. If tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{k}$ cuts the x-axis and y-axis at p and q respectively then, prove that $p+q=k$ is the only condition when it is possible.

Q09. Find the intervals in which the following function f is (a) increasing (b) decreasing:

$$f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x},\ 0\le x\le 2\pi.$$

Q10. Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Answers of Applications Of Derivatives

TEST 01

Q02. $\frac{1}{a}-\frac{1}{b}=\frac{1}{\alpha}-\frac{1}{\beta}$

Q04. $a=1,\ b=2$

Q06. Increasing : $(2,3)$; decreasing : $(3,\infty)$ or Increasing : $(1/e,\infty)$; decreasing : $(0,1/e)$

Q09. $\left(\frac{h^2}{4}\right)$ sq.units

Q10. Square : $\frac{20}{4+3\sqrt{3}}m$; Triangle : $\frac{60\left(1+\sqrt{3}\right)}{4+3\sqrt{3}}m$.

TEST 02

Q01. $\left(\frac{1}{4},\frac{25}{8}\right)$

Q02. $x+y=3$

Q04. (a) Increasing : $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right)$; decreasing : $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$

Q04. (b) Increasing : $(-\infty,2)\cup(3,\infty)$; decreasing : $(2,3)$

Q05. 0.192

Q06. $\frac{4\pi}{3\sqrt{3}}R^3$ cubic units

Q07. $\frac{1}{48}cm/sec$

Q09. Increasing : $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$; decreasing : $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$.

Any query regarding any question in this test? Write to me on ***theopgupta@gmail.com***