Derivatives Of Various Types

TEST – 01

Q01. Given that $f(x)=\left(\frac{3+x}{1+x}\right)^{2+3x}$ then, find $f'(0)$.

Q02. Differentiate $2\tan^{-1}\left(\frac{\sqrt{2}x}{1-x^2}\right)+\log\left(\frac{1+\sqrt{2}x+x^2}{1-\sqrt{2}x+x^2}\right)$ with respect to *x*.

OR If $x=a(\sin\theta-\theta\cos\theta)$, $y=a(\cos\theta+\theta\sin\theta)$ then, find $\left.\frac{d^2y}{dx^2}\right]_{at\ \theta=\pi/4}$.

Q03. If $y=x\log\left(\frac{x}{a+bx}\right)$ then, write the expression for $\frac{d^2y}{dx^2}$.

Q04. Differentiate the followings *w.r.t. x*:

(a) $\tan^{-1}\left(\frac{\sqrt{x}+x}{1-x^{3/2}}\right)$ (b) $\sin^{-1}\left(\frac{x^2}{\sqrt{x^4+a^4}}\right)$.

OR Differentiate $\cos(2\sin^{-1}x)+\sin(2\sin^{-1}x)$ with respect to *x*.

Q05. Differentiate $\cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

Q06. If $y=\sin\sin x$ then, show that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2x=0$.

OR If $\sin y=x\sin(\alpha+y)$ then, prove that $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\frac{\sin^2(\alpha+y)}{\sin\alpha}$.

Q07. Verify Rolle's Theorem for the function $f(x)=\log\left(\frac{x^2+ab}{x(a+b)}\right)$ on $[a, b]$.

Q08. Write the derivative of: (a) $10^{5\log_{10}x}$ (b) $\log_a x^2$ (c) $\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$.

Q09. (a) Find the derivative of $f(\log x)$ if $f(x)=\log x$.

(b) If $f(x)=\sin^{-1}\left[\frac{12x+5\sqrt{1-x^2}}{13}\right]$ then, find $f'(x)$.

Q10. Given that $y=e^{ax}\sin bx$ then, prove that $y_2-2ay_1+(a^2+b^2)y=0$.

OR Differentiate $2\tan^{-1}\left(\operatorname{cosec}\tan^{-1}x-\tan\cot^{-1}x\right)$ with respect to *x*.

TEST – 02

Q01. Differentiate the followings with respect to x:
(a) $\sqrt{e^{\sqrt{x}}}$ (b) $\tan^{-1}\sqrt{x}$.

Q02. If $(x+y)^{45}=x^{28}y^{17}$ then, prove that $\frac{dy}{dx}=\frac{y}{x}$. Hence find $\frac{d^2y}{dx^2}$ as well.

Q03. If $x=ae^{\theta}(\sin\theta-\cos\theta)$ and $y=ae^{\theta}(\sin\theta+\cos\theta)$ then, show that $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$ is 1.

Q04. Differentiate $(\log x)^x+x^{\log x}$ with respect to x.

Q05. Given that $y=\tan^{-1}\left(\frac{1-x}{1+x}\right)+\tan^{-1}\left(\frac{2+x}{1-2x}\right)$. Write the value of $\frac{dy}{dx}$.

Q06. If $y=\left(\tan^{-1}x\right)^2$ then, show that $\left(x^2+1\right)^2y_2+2x\left(x^2+1\right)y_1=2$.
OR Differentiate $\log\left(x+\sqrt{x^2-1}\right)$ with respect to $\log\left(x+\sqrt{x^2+1}\right)$.

Q07. Find the derivative of $\cos^{-1}\left(x\sqrt{1-x^4}-x^2\sqrt{1-x^2}\right)$ with respect to x.

Q08. Differentiate $\frac{x\sin x}{1+\sin x}+\log\sqrt{\frac{1+\cos^2x}{1-e^{2x}}}$ *w.r.t. x*.
OR If $y=\sqrt{\frac{1-x}{1+x}}$ then, prove that $\left(1-x^2\right)\frac{dy}{dx}+y=0$.

Q09. If $xy\log(x+y)=1$ then, prove that $\frac{dy}{dx}=-\frac{y(x^2y+x+y)}{x(xy^2+x+y)}$.

Q10. Differentiate y with respect to x if $\tan y=\frac{\sqrt{1+t^2}-1}{t}$ and $\tan x=\frac{2t}{1-t^2}$.

Answers of Derivatives Of Various Types
TEST – 01

Q01. $3(9\log 3-4)$ Q02. $\frac{4\sqrt{2}}{1+x^4}$ or $-\frac{8\sqrt{2}}{a\pi}$ Q03. $\frac{1}{x}\left[\frac{a}{a+bx}\right]^2$

Q04.(a) $\frac{1}{2\sqrt{x}(1+x)}+\frac{1}{1+x^2}$ (b) $\frac{2a^2x}{a^4+x^4}$ or $\frac{2(1-2x^2)}{\sqrt{1-x^2}}-4x$ Q05. $-\frac{1}{4}$

Q07. $c\in\sqrt{ab}$ Q08.(a) $5x^4$ (b) $\frac{2\log_a e}{x}$ (c) $\sec x$

Q09.(a) $\frac{1}{x\log x}$ (b) $\frac{1}{\sqrt{1-x^2}}$ Q10. $\frac{1}{1+x^2}$.

TEST 02

Q01.(a) $\frac{e^{\sqrt{x}}}{4\sqrt{x e^{\sqrt{x}}}}$ (b) $\frac{1}{2\sqrt{x}(1+x)}$ Q02. 0 Q04. $(\log x)^x\left(\frac{1}{\log x}+\log\log x\right)+x^{\log x}\left(\frac{2\log x}{x}\right)$

Q05. 0 Q06. $\sqrt{\frac{x^2+1}{x^2-1}}$ Q07. $\frac{2x}{\sqrt{1-x^4}}-\frac{1}{\sqrt{1-x^2}}$

Q08. $\frac{x\cos x}{(1+\sin x)^2}+\frac{\sin x}{1+\sin x}+\frac{1}{2}\left[\frac{2e^{2x}}{1-e^{2x}}-\frac{\sin 2x}{1+\cos^2x}\right]$ Q10. $\frac{1}{4}$.

 # Any query regarding any question in this test? Write to me on ***theopgupta@gmail.com***