Matrices & Determinants

TEST – 01

Q01. Find the value of x, $x \in Z$ such that $\begin{bmatrix} x & 4 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} x & 4 & -1 \end{bmatrix}' = 0$.

Q02. Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of symmetric and a skew-symmetric matrix.

Q03. If a, b, c are in arithmetic progression, prove that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+4 & x+b \\ x+3 & x+6 & x+c \end{vmatrix} = 0$.

OR Solve: $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$.

Q04. Solve: $\frac{2}{a} + \frac{3}{b} + \frac{10}{c} = 4, \frac{4}{a} - \frac{6}{b} + \frac{5}{c} = 1, \frac{6}{a} + \frac{9}{b} + -\frac{20}{c} = 2$.

OR Using properties of determinants, prove that: $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$.

Q05. Construct a 2×2 matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$, where $a_{ij} = \frac{(3i-j)^2}{2}$.

Q06. Find the value of p and q such that $A^2 + pI = qA$, where $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$.

Q07. Prove that: $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$.

Q08. Evaluate: $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

OR In XII class examination, 25 students from school A and 35 students from school B appeared. Only 20 students from each school could get through the examination. Out of them, 15 students from school A and 10 students from school B secured full marks. Write down this information in matrix from.

Q09. Find A^{-1}, if $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$. Also show that $A^{-1} = \frac{A^2 - 3I}{2}$.

Q10. Determine a matrix X such that $2A + B + X = 0$, where $A = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 1 & 5 \end{pmatrix}$.

TEST – 02

Q01. Find the equation of the line joining the points (1, 2) and (3, 6) using determinants.

Q02. For what value of x, the matrix $\begin{pmatrix} 5-x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular?

Q03. If matrix A is of order 3×3 and $|A|=9$, find the value of $|A.adjA|$.

Q04. Given a square matrix A of order 3×3, it is provided that $|3A|=k|A|$. Write the value of k.

OR What is the value of $|3I_3|$, where I_3 is the identity matrix of order 3?

Q05. If A is a matrix of order 2×3 and B is a matrix of order 3×5, then write the order of matrix $(AB)^T$.

Q06. Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. It is found that the shopkeepers A, B, C are using (20,30,40), (30,40,20,), (40,20,30) polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent ₹250, ₹270 & ₹200 on these carry bags respectively. Find the cost of each carry bags using matrices. Keeping in mind the social & environmental conditions, which shopkeeper is better? Why?

Q07. If $A=\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2-5A-14I=0$. Hence or, otherwise find a matrix B such that $BA=I$.

Q08. Using properties of determinants, prove the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3 \qquad \text{OR} \qquad \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

Q09. Using induction, prove that $A^n=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, $n\in N$ if $A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$.

OR Using elementary transformations, find the inverse of $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$.

Q10. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}, B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ then, find AB. Use this to solve the given system of equations: $2x-y+z=-1$, $2y-z-x=4$, $x-y+2z=-3$.

OR If $A=\begin{bmatrix} 2 & 0 & -1 \\ 1 & 2 & 3 \\ 2 & 2 & -1 \end{bmatrix}$ then compute A^{-1}. Hence solve: $2x-z=4, x+2y+3z=0, 2x+2y-z=2$.

Answers of Matrices & Determinants

TEST – 01

Q01. $x = -4$ Q02. $\begin{bmatrix} 4 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & 9/2 \\ 1 & -9/2 & 0 \end{bmatrix}$ Q03. $\frac{2}{3}, \frac{11}{3}$

Q04. $a = 2,\ b = 3,\ c = 5$ Q05. $\begin{bmatrix} 2 & 1/2 \\ 25/2 & 8 \end{bmatrix}$ Q06. $p = q = 8$

Q08. $\begin{bmatrix} ac + bd + a^2 + b^2 + c^2 + d^2 \end{bmatrix}$

OR Given information is expressed in matrix:

	School A	School B
Appeared →	25	35
Got through exam →	20	20
Secured full marks →	15	10

Q09. $\frac{1}{2}\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ Q10. $X = \begin{pmatrix} -1 & -2 \\ -7 & -13 \end{pmatrix}$.

TEST – 02

Q01. $2x - y = 0$ Q02. $x = 3$ Q03. 729 Q04. 27 OR 27

Q05. 5×2 Q06. Polythene = ₹1, Handmade bag = ₹5, Newspaper's envelope = ₹2. Shopkeeper A is better for environmental conditions. As he is using least no. of polythene. Shopkeeper B is also better for social conditions as he is using handmade bags (prepared by prisoners). Shopkeeper C is better too as the newspaper's envelope used by him give employment to some people.

Q07. $\frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}$ Q09. $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$

Q10. $AB = 4\,\mathrm{I};\ x = 1, y = 2, z = -1$ or $A^{-1} = \frac{1}{14}\begin{bmatrix} 8 & 2 & -2 \\ -7 & 0 & 7 \\ 2 & 4 & -4 \end{bmatrix};\ x = 2, y = -1, z = 0$.

 # Any query regarding any question in this test? Write to me on ***theopgupta@gmail.com***