Complete Study Guide & Notes On
INVERSE TRIGONOMETRIC FUNCTIONS

A Formulae Guide By OP Gupta (Indira Award Winner)

The essence of Mathematics is not to make simple things complicated,
but to make complicated things simple!

IMPORTANT TERMS, DEFINITIONS & RESULTS

❖ A list of formulae for **Trigonometric Functions** (class XI) has been added for reference at the end of this formulae guide.

01. Basic Introduction: A function $f : \mathrm{A} \to \mathrm{B}$ is said to be invertible if f is bijective (*i.e., one-one and onto*). The inverse of the function *f* is denoted by $f : \mathrm{B} \to \mathrm{A}$ such that $f^{-1}(y) = x$ if $f(x) = y$, $\forall\, x \in \mathrm{A},\ y \in \mathrm{B}$. As trigonometric functions are many-one so, their inverse doesn't exist. But they become one-one onto by restricting their domains. Therefore, inverse of trigonometric functions are defined with restricted domains. In fact, in the discussion below we have used all the restrictions required so that the inverse of the concerned trigonometric functions do exist. If these restrictions are removed, the terms will represent **inverse trigonometric relations** and not the functions. Note that the inverse trigonometric functions are also called as **Inverse Circular Functions**.

02. List of Formulae and their proofs for Inverse Trigonometric Functions:

A.

a) $\sin^{-1}(x) = \mathrm{cosec}^{-1}\left(\dfrac{1}{x}\right),\ x \in [-1,1]$

b) $\mathrm{cosec}^{-1}(x) = \sin^{-1}\left(\dfrac{1}{x}\right),\ x \in (-\infty,-1] \cup [1,\infty)$

c) $\cos^{-1}(x) = \sec^{-1}\left(\dfrac{1}{x}\right),\ x \in [-1,1]$

d) $\sec^{-1}(x) = \cos^{-1}\left(\dfrac{1}{x}\right),\ x \in (-\infty,-1] \cup [1,\infty)$

e) $\tan^{-1}(x) = \begin{cases} \cot^{-1}\left(\dfrac{1}{x}\right), & x > 0 \\ -\pi + \cot^{-1}\left(\dfrac{1}{x}\right), & x < 0 \end{cases}$

f) $\cot^{-1}(x) = \begin{cases} \tan^{-1}\left(\dfrac{1}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\dfrac{1}{x}\right), & x < 0 \end{cases}$

<u>**PROOF**</u> **a)** Let $\sin^{-1}(x) = \theta$ then, $\sin\theta = x$

$\Rightarrow \mathrm{cosec}\,\theta = \dfrac{1}{x}$ $\qquad \Rightarrow \theta = \mathrm{cosec}^{-1}\left(\dfrac{1}{x}\right)$

$\therefore \qquad \sin^{-1} x = \mathrm{cosec}^{-1}\left(\dfrac{1}{x}\right)$ **[H.P.]**

Other results can be proved in the same way!

B.

a) $\sin^{-1}(-x) = -\sin^{-1} x,\ x \in [-1,1]$

b) $\cos^{-1}(-x) = \pi - \cos^{-1} x,\ x \in [-1,1]$

c) $\tan^{-1}(-x) = -\tan^{-1} x,\ x \in \mathrm{R}$

d) $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1} x,\ |x| \ge 1$

e) $\sec^{-1}(-x) = \pi - \sec^{-1} x,\ |x| \ge 1$

f) $\cot^{-1}(-x) = \pi - \cot^{-1} x,\ x \in \mathrm{R}$

<u>**PROOF**</u> **b)** Let $\cos^{-1}(-x) = \theta$ then, $\cos\theta = -x$

$\Rightarrow -\cos\theta = x \quad \Rightarrow \cos(\pi - \theta) = x \qquad \Rightarrow \cos^{-1} x = \pi - \theta$

$\Rightarrow \theta = \pi - \cos^{-1} x \qquad \Rightarrow \cos^{-1}(-x) = \pi - \cos^{-1} x$ **[H.P.]**

Similarly, we can prove other results!

C.

a) $\sin^{-1}(\sin x) = x,\ -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

b) $\cos^{-1}(\cos x) = x,\ 0 \le x \le \pi$

c) $\tan^{-1}(\tan x) = x,\ -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

d) $\mathrm{cosec}^{-1}(\mathrm{cosec}\, x) = x,\ -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}, x \ne 0$

e) $\sec^{-1}(\sec x) = x,\ 0 \le x \le \pi,\ x \ne \dfrac{\pi}{2}$

f) $\cot^{-1}(\cot x) = x,\ 0 < x < \pi$

PROOF a) Let $\sin^{-1}(x)=\theta \quad ...(i)$

then, $\sin\theta = x \quad ...(ii)$

Substituting the value of x in (i) from (ii) we get,

$\sin^{-1}\sin\theta=\theta$ (Replacing θ by x)

$\therefore \quad \sin^{-1}\sin x = x$ **[H.P.]**

For other results we can proceed similarly!

D. **a)** $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\ x\in[-1,1]$

b) $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\ x\in \mathrm{R}$

c) $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2},\ |x|\geq 1$ *i.e.*, $x\leq -1$ or $x\geq 1$

PROOF a) Let $\sin^{-1}(x)=\theta$ then, $\sin\theta=x$

$\Rightarrow \cos\left(\frac{\pi}{2}-\theta\right)=x \qquad \Rightarrow \frac{\pi}{2}-\theta=\cos^{-1}x \qquad \Rightarrow \frac{\pi}{2}=\cos^{-1}x+\theta$

$\Rightarrow \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\ -1\leq x\leq 1$ **[H.P.]**

Similarly, proceed for other results!

E. **a)** $\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right]$

b) $\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left[xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right]$

c) $$\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\left(\frac{x+y}{1-xy}\right), & xy<1\\ \pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right), & x>0,\ y>0,\ xy>1\\ -\pi+\tan^{-1}\left(\frac{x+y}{1-xy}\right), & x<0,\ y<0,\ xy>1\end{cases}$$

d) $$\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\left(\frac{x-y}{1+xy}\right), & xy>-1\\ \pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & x>0,\ y<0,\ xy<-1\\ -\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & x<0,\ y>0,\ xy<-1\end{cases}$$

e) $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

PROOF a) Let $\sin^{-1}x=\theta$ and $\sin^{-1}y=\beta$. Then, $\sin\theta=x$ and $\sin\beta=y$.

Now $\sin(\theta+\beta)=\sin\theta\cos\beta+\cos\theta\sin\beta$

$\Rightarrow \qquad =\sin\theta\sqrt{1-\sin^2\beta}+\sqrt{1-\sin^2\theta}\sin\beta$

$\Rightarrow \sin(\theta+\beta)=x\sqrt{1-y^2}+y\sqrt{1-x^2}$

$\Rightarrow \theta+\beta=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$

$\therefore \quad \sin^{-1}x+\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]$ **[H.P.]**

b) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left[xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right]$

Do yourself. Proceed as in (a).

c) Let $\tan^{-1} x = \theta$ and $\tan^{-1} y = \beta$. Then, $\tan\theta = x$ and $\tan\beta = y$.

Now $\tan(\theta+\beta) = \dfrac{\tan\theta + \tan\beta}{1-\tan\theta\tan\beta} = \dfrac{x+y}{1-xy}$

$$\therefore \quad \theta+\beta = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\frac{x+y}{1-xy}\right]$$

For $x>0$, $\tan^{-1} x$ will be a positive angle and for $y>0$, $\tan^{-1} y$ will also be a positive angle. Therefore, LHS of (c) will be a positive angle and hence RHS should also be a positive angle.

__Case I__ When $x>0, y>0$ and $xy<1$ then $\dfrac{x+y}{1-xy}$ is positive.

So, $\tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ will be a positive angle.

Hence, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ **[H.P.]**

__Case II__ When $x>0, y>0$ and $xy>1$ then $\dfrac{x+y}{1-xy}$ is negative.

So, $\tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ will be a negative angle. Therefore we add π to make it positive and balanced.

Hence, $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ **[H.P.]**

__Case III__ When $x<0, y<0$ and $xy>1$ then $\dfrac{x+y}{1-xy}$ is positive.

So, $\tan^{-1} x + \tan^{-1} y$ will be a negative angle and $\tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ will be a positive angle. Therefore to balance it we will be adding $-\pi$.

Hence, $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1}\left[\dfrac{x+y}{1-xy}\right]$ **[H.P.]**

d) Let $\tan^{-1} x = \theta$ and $\tan^{-1} y = \beta$. Then, $\tan\theta = x$ and $\tan\beta = y$.

Now $\tan(\theta-\beta) = \dfrac{\tan\theta - \tan\beta}{1+\tan\theta\tan\beta} = \dfrac{x-y}{1+xy}$

$$\therefore \quad \theta-\beta = \tan^{-1}\left[\frac{x-y}{1+xy}\right]$$

$$\Rightarrow \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left[\frac{x-y}{1+xy}\right] \qquad \ldots(A)$$

__Case I__ When $x>0, y>0$ and $xy \ge -1$ then $\dfrac{x-y}{1+xy}$ is positive (or negative depending upon the absolute value of the angles *x* and *y*). Also if $x>0, y>0$ then, $\theta,\beta \in \left(0,\dfrac{\pi}{2}\right)$. So $\theta-\beta \in \left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ i.e.,

$\tan^{-1}x - \tan^{-1}y \in \left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$.

Hence, $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\left[\dfrac{x-y}{1+xy}\right], xy\ge -1$ [By using (A)] **[H.P.]**

Case II When $x>0, y<0$ and $xy<-1$ then $\dfrac{x-y}{1+xy}$ is negative. Also if $x>0, y<0$ then, $\theta\in\left(0,\dfrac{\pi}{2}\right),\beta\in\left(-\dfrac{\pi}{2},0\right)$ i.e., $\theta\in\left(0,\dfrac{\pi}{2}\right),-\beta\in\left(0,\dfrac{\pi}{2}\right)$. So $\theta-\beta\in(0,\pi)$ i.e., $\tan^{-1}x-\tan^{-1}y\in(0,\pi)$.

As $\tan^{-1}\left[\dfrac{x-y}{1+xy}\right]$ is a negative angle. Therefore we add π in RHS of (A) to make it positive and balanced.

Hence, $\tan^{-1}x-\tan^{-1}y=\pi+\tan^{-1}\left[\dfrac{x-y}{1+xy}\right], x>0,\ y<0$ and $xy<-1$ **[H.P.]**

Case III When $x<0, y>0$ and $xy<-1$ then $\dfrac{x-y}{1+xy}$ is positive. Also if $x<0, y>0$ then, $\theta\in\left(-\dfrac{\pi}{2},0\right),\beta\in\left(0,\dfrac{\pi}{2}\right)$ i.e., $\theta\in\left(-\dfrac{\pi}{2},0\right),-\beta\in\left(-\dfrac{\pi}{2},0\right)$. So $\theta-\beta\in(-\pi,0)$ i.e., $\tan^{-1}x-\tan^{-1}y\in(-\pi,0)$.

So, $\tan^{-1}x-\tan^{-1}y$ will be a negative angle and $\tan^{-1}\left[\dfrac{x-y}{1+xy}\right]$ will be a positive angle. Therefore to balance it we will be adding $-\pi$ in RHS of (A).

Hence, $\tan^{-1}x-\tan^{-1}y=-\pi+\tan^{-1}\left[\dfrac{x-y}{1+xy}\right], x<0,\ y>0$ and $xy<-1$ **[H.P.]**

F. **a)** $2\tan^{-1}x=\sin^{-1}\left(\dfrac{2x}{1+x^2}\right), |x|\le 1$

b) $2\tan^{-1}x=\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right), x\ge 0$

c) $2\tan^{-1}x=\tan^{-1}\left(\dfrac{2x}{1-x^2}\right), -1<x<1$

PROOF a) Let $\tan^{-1}x=\theta$ then, $\tan\theta=x$.

As $\sin 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$ $\Rightarrow \sin 2\theta=\dfrac{2x}{1+x^2}$

i.e., $2\theta=\sin^{-1}\dfrac{2x}{1+x^2}$ $\Rightarrow 2\tan^{-1}x=\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ **[H.P.]**

Other results can also be proved in the same way!

03. Principal Value: Numerically *smallest angle* is known as the principal value.

Finding the principal value: For finding the principal value, following algorithm can be followed–

STEP1– Firstly, draw a trigonometric circle and mark the quadrant in which the angle may lie.

STEP2– Select anticlockwise direction for 1st and 2nd quadrants and clockwise direction for 3rd and 4th quadrants.

STEP3– Find the angles in the first rotation.

STEP4– Select the numerically least (magnitude wise) angle among these two values. The angle thus found will be the principal value.

STEP5– In case, two angles one with positive sign and the other with the negative sign qualify for the numerically least angle then, it is the convention to select the angle with positive sign as principal value.

*The principal value is **never** numerically greater than π.*

04. Table demonstrating domains and ranges of Inverse Trigonometric functions:

Inverse Trigonometric Functions *i.e.,* $f(x)$	Domain/ Values of x	Range/ Values of $f(x)$
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\text{cosec}^{-1} x$	$R-(-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$\sec^{-1} x$	$R-(-1, 1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$

Discussion about the range of inverse circular functions other than their respective principal value branch

We know that the domain of sine function is the set of real numbers and range is the closed interval [–1, 1]. If we restrict its domain to $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ etc. then, it becomes bijective with the range [–1, 1]. So, we can define the inverse of sine function in each of these intervals. Hence, all the intervals of $\sin^{-1}$ function, **except principal value branch** (here except of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for $\sin^{-1}$ function) are known as the **range of $\sin^{-1}$ other than its principal value branch**. The same discussion can be extended for other inverse circular functions. (Refer Q16 in Mathematicia Vol.1 By OP Gupta)

05. To simplify inverse trigonometrical expressions, following substitutions can be considered:

Expression	Substitution
a^2+x^2 or $\sqrt{a^2+x^2}$	$x = a\tan\theta$ or $x = a\cot\theta$
a^2-x^2 or $\sqrt{a^2-x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
x^2-a^2 or $\sqrt{x^2-a^2}$	$x = a\sec\theta$ or $x = a\ \text{cosec}\,\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2\cos 2\theta$
$\sqrt{\frac{x}{a-x}}$ or $\sqrt{\frac{a-x}{x}}$	$x = a\sin^2\theta$ or $x = a\cos^2\theta$
$\sqrt{\frac{x}{a+x}}$ or $\sqrt{\frac{a+x}{x}}$	$x = a\tan^2\theta$ or $x = a\cot^2\theta$

Note the followings and keep them in mind:

➲ *The symbol* $\sin^{-1}x$ *is used to denote the* ***smallest angle*** *whether positive or negative, such that the sine of this angle will give us x. Similarly* $\cos^{-1}x$, $\tan^{-1}x$, $\text{cosec}^{-1}x$, $\sec^{-1}x$, *and* $\cot^{-1}x$ *are defined.*

➲ *You should note that* $\sin^{-1}x$ *can be written as* ***arcsinx***. *Similarly other Inverse Trigonometric Functions can also be written as arccosx, arctanx, arcsecx etc.*

➲ *Also* ***note that*** $\mathbf{\sin^{-1}x}$ *(and similarly other Inverse Trigonometric Functions)* ***is entirely different from*** $(\mathbf{\sin x})^{-1}$. *In fact,* $\sin^{-1}x$ *is the measure of an angle in Radians whose sine is x whereas* $(\sin x)^{-1}$ *is* $\dfrac{1}{\sin x}$ *(which is obvious as per the* ***laws of exponents****).*

➲ *Keep in mind that these inverse trigonometric relations are* ***true only in their domains*** *i.e., they are valid only for some values of 'x' for which inverse trigonometric functions are well defined!*

Check out NCERT Textbook Part I for the **Graphs** of Inverse Trigonometric Functions.

❖ Trigonometric Formulae (Only For Reference):

◆ ***Relation between trigonometric ratios***

a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ *b)* $\tan\theta = \dfrac{1}{\cot\theta}$ *c)* $\tan\theta.\cot\theta = 1$

d) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ *e)* $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ *f)* $\sec\theta = \dfrac{1}{\cos\theta}$

◆ ***Trigonometric identities***

a) $\sin^2\theta + \cos^2\theta = 1$

b) $\sec^2\theta = 1 + \tan^2\theta$

c) $\text{cosec}^2\theta = 1 + \cot^2\theta$

◆ ***Addition / subtraction formulae & some related results***

a) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

b) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

c) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

d) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

f) $\cot(A \pm B) = \dfrac{\cot B\cot A \mp 1}{\cot B \pm \cot A}$

◆ ***Multiple angle formulae involving 2A & 3A***

a) $\sin 2A = 2\sin A\cos A$

b) $\sin A = 2\sin\dfrac{A}{2}\cos\dfrac{A}{2}$

c) $\cos 2A = \cos^2 A - \sin^2 A$

d) $\cos A = \cos^2\dfrac{A}{2} - \sin^2\dfrac{A}{2}$

e) $\cos 2A = 2\cos^2 A - 1$

f) $2\cos^2 A = 1 + \cos 2A$

g) $\cos 2A = 1 - 2\sin^2 A$

h) $2\sin^2 A = 1 - \cos 2A$

i) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

j) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

k) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

l) $\sin 3A = 3\sin A - 4\sin^3 A$

m) $\cos 3A = 4\cos^3 A - 3\cos A$

n) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

◆ ***Transformation of sums / differences into products & vice-versa***

a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

b) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

d) $\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

e) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

f) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

g) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

h) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

◆ ***Relations in Different Measures of Angle***

➲ $\text{Angle in Radian Measure} = (\text{Angle in Degree Measure}) \times \dfrac{\pi}{180}$

➲ $\text{Angle in Degree Measure} = (\text{Angle in Radian Measure}) \times \dfrac{180}{\pi}$

➲ $\theta\ (\text{in radian measure}) = \dfrac{l}{r}$

Also followings are of importance as well:

➲ $1\ \textit{Right angle} = 90^o$

➲ $1^o = 60'$, $1' = 60''$

➲ $1^o = \dfrac{\pi}{180} = 0.01745\ \textit{radians}\,(\textit{Approx.})$

➲ $1\ \textit{radian} = 57^o17'45''$ *or 206265 seconds*.

◆ ***General Solutions***

a) $\sin x = \sin y \;\Rightarrow\; x = n\pi + (-1)^n y$, *where* $n \in Z$.

b) $\cos x = \cos y \;\Rightarrow\; x = 2n\pi \pm y$, *where* $n \in Z$.

c) $\tan x = \tan y \;\Rightarrow\; x = n\pi + y$, *where* $n \in Z$.

◆ ***Relation in Degree & Radian Measures***

Angles in Degree	0^o	30^o	45^o	60^o	90^o	180^o	270^o	360^o
Angles in Radian	0^c	$\left(\frac{\pi}{6}\right)^c$	$\left(\frac{\pi}{4}\right)^c$	$\left(\frac{\pi}{3}\right)^c$	$\left(\frac{\pi}{2}\right)^c$	$(\pi)^c$	$\left(\frac{3\pi}{2}\right)^c$	$(2\pi)^c$

➷ *In actual practice, we omit the exponent '**c**' and instead of writing π^c we simply write π and similarly for others.*

For the complete discussion of the Trigonometric Functions, please refer to the **FORMULAE GUIDE Of TRIGONOMETRIC FUNCTIONS of Class XI**.

❖ For the values of Trigonometric Ratios at Standard Angles (i.e, $0^o, 30^o, 45^o, 60^o$ and 90^o), check the following page.

◆ *Trigonometric Ratio of Standard Angles*

Degree / Radian	0°	30°	45°	60°	90°
T – Ratios	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

◆ *Trigonometric Ratios of Allied Angles*

Angles $(\rightarrow)$ / **T- Ratios** $(\downarrow)$	$\frac{\pi}{2}-\theta$	$\frac{\pi}{2}+\theta$	$\pi-\theta$	$\pi+\theta$	$\frac{3\pi}{2}-\theta$	$\frac{3\pi}{2}+\theta$	$2\pi-\theta$ **OR** $-\theta$	$2\pi+\theta$
sin	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
cos	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
tan	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
cot	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$
sec	$\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$	$\text{cosec}\theta$	$\sec\theta$	$\sec\theta$
cosec	$\sec\theta$	$\sec\theta$	$\text{cosec}\theta$	$-\text{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\theta$	$\text{cosec}\theta$

◆ *Domain and Range of Trigonometric Functions*

T- Functions	**Domain**	**Range**
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$
$\tan x$	$\{x \in R : x \neq (2n+1)\pi/2, n \in Z\}$	R
$\cot x$	$\{x \in R : x \neq n\pi, n \in Z\}$	R
$\text{cosec}\, x$	$\{x \in R : x \neq n\pi, n \in Z\}$	$R-(-1, 1)$
$\sec x$	$\{x \in R : x \neq (2n+1)\pi/2, n \in Z\}$	$R-(-1, 1)$

Any queries and/or suggestion(s), please write to me at
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Please mention your details : Name, Student/Teacher/Tutor, School/Institution, Place, Contact No. (if you wish)